Mathematics — Solved Paper 2019

SECTION A (40 Marks)

*(Answer **all** questions from this Section)*

***Question* 1 :**

(a) Solve the following inequation and write down the solution set : [3]

$$11x - 4 < 15x + 4 \leq 13x + 14, \ x \in W$$

Represent the solution on a real number line.

Solution :

$11x - 4 < 15x + 4 \leq 13x + 14, \ x \in W$

$\Rightarrow \quad 11x - 4 < 15x + 4 \quad$ and $\quad 15x + 4 \leq 13x + 14$

$\Rightarrow \quad 11x - 15x < 4 + 4 \quad$ and $\quad 15x - 13x \leq 14 - 4$

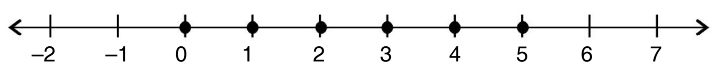
$\Rightarrow \quad -4x < 8 \quad$ and $\quad 2x \leq 10$

$\Rightarrow \quad x > -2 \quad$ and $\quad x \leq 5$

Hence, $\quad -2 < x \leq 5, \ x \in W$

$\Rightarrow$ **Solution set = {0, 1, 2, 3, 4, 5}** **Ans.**

Solution on the real number line is :



Ans.

(b) A man invests ₹ 4,500 in shares of a company which is paying 7·5% dividend. If ₹ 100 shares are available at a discount of 10% [3]

Find :

(i) number of shares he puchases. (ii) his annual income.

Solution :

$\because$ N.V. of each share = ₹ 100 and discount = 10%

$\therefore$ M.V. of each share = ₹ 100 – 10% of ₹ 100 = ₹ 90

(i) **Number of shares puchased** $= \dfrac{\text{Investment}}{\text{M.V. of each share}}$

$= \dfrac{₹\,4{,}500}{₹\,90}$ **= 50** **Ans.**

(ii) $\because$ N.V. of each share = ₹ 100 and dividend = 7·5%

$\Rightarrow$ Dividend on each share = 7·5% of ₹ 100

= ₹ 7·50

His annual income = Dividend on 50 shares

= 50 × ₹ 7·50 = **₹ 375** **Ans.**

Direct Method,

His annual income = N.V. of each share × no. of shares × dividend% on each share

= ₹ 100 × 50 × 7·5% = **₹ 375** **Ans.**

(c) In a class of 40 students, marks obtained by the students in a class test (out of 10) are given below : [4]

Marks	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
No. of students	**1**	**2**	**3**	**3**	**6**	**10**	**5**	**4**	**3**	**3**

Calculate the following for the given distribution :

(i) median (ii) mode.

Solution :

Marks (x)	1	2	3	4	5	6	7	8	9	10
No. of students (f)	1	2	3	3	6	10	5	4	3	3
Cumulative frequency ($c.f.$)	1	3	6	9	15	25	30	34	37	40

(i) **Median** $= \frac{1}{2}\left[\frac{40}{2}^{\text{th}} \text{ term} + \left(\frac{40}{2}+1\right)^{\text{th}} \text{ term}\right]$

$= \frac{1}{2}\left[20^{\text{th}} \text{ term} + 21^{\text{st}} \text{ term}\right]$

$= \frac{1}{2}[6 + 6] = \frac{1}{2} \times 12 = \mathbf{6}$ **Ans.**

(ii) **Mode** = Marks with highest frequency **= 6** **Ans.**

***Question* 2 :**

(a) Using the factor theorem, show that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$. Hence factorise the polynomial completely. [3]

Solution :

$x - 2 = 0 \Rightarrow x = 2$

Value of $x^3 + x^2 - 4x - 4$ for $x = 2$

$= (2)^3 + (2)^2 - 4 \times 2 - 4$

$= 8 + 4 - 8 - 4 = 0$

$\Rightarrow$ **$(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$** **Ans.**

Now, $x^3 + x^2 - 4x - 4$

$= (x - 2)(x^2 + 3x + 2)$

$= (x - 2)(x^2 + 2x + x + 2)$

$= (x - 2)[x(x + 2) + 1(x + 2)]$

$= \mathbf{(x - 2)(x + 2)(x + 1)}$

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\
\hline
x - 2 & x^3 + x^2 - 4x - 4 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & 3x^2 - 4x \\
 & 3x^2 - 6x \\
 & - \quad + \\
\hline
 & 2x - 4 \\
 & 2x - 4 \\
 & - \quad + \\
\hline
 & \times \\
\hline
\end{array}
$$

(b) Prove that : (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ) = 1. **[3]**

Solution :

$$\textbf{L.H.S.} = \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\cos\theta\sin\theta} = 1 = \textbf{R.H.S.}$$ **Hence Proved**

(c) In an Arithmetic Progression (A.P.) the fourth and sixth terms are 8 and 14 respectively. Find the : **[4]**

(i) first term (ii) common difference (iii) sum of the first 20 terms.

Solution :

Given, the fourth term of A.P. = 8

⇒ $a + 3d = 4$ I

Also, the sixth term of the same A.P. = 14

⇒ $a + 5d = 14$ II

On solving equations I and II, we get : $a = -11$ and $d = 5$

∴ (i) **first term** = a = **– 11** **Ans.**

(ii) **common difference** = d = **5** **Ans.**

(iii) $S_n = \frac{n}{2}[2a + (n-1)d]$

⇒ **Sum of first 20 terms** = $\frac{20}{2}[2 \times -11 + (20-1) \times 5]$

$= 10[-22 + 95]$

$= 10 \times 73 =$ **730** **Ans.**

***Question* 3 :**

(a) Simplify : $\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix} + \cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$. **[3]**

Solution :

$$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix} + \cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$$

$$= \begin{bmatrix}\sin^2 A & -\sin A\cos A\\ \sin A\cos A & \sin^2 A\end{bmatrix} + \begin{bmatrix}\cos^2 A & \cos A\sin A\\ -\cos A\sin A & \cos^2 A\end{bmatrix}$$

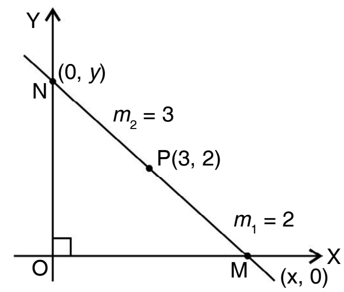
$$= \begin{bmatrix}\sin^2 A + \cos^2 A & -\sin A\cos A + \cos A\sin A\\ \sin A\cos A - \cos A\sin A & \sin^2 A + \cos^2 A\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$ **Ans.**

(b) M and N are two points on the X axis and Y axis respectively. P(3, 2) divides the line segment MN in the ratio 2 : 3. Find :

(i) the coordinates of M and N. **(ii) slope of the line MN.** **[3]**

Solution :



Given PM : PN = 2 : 3 = $m_1 : m_2$

(i) Let M = $(x, 0)$ and N = $(0, y)$

For point M :

$$P(x) = \frac{m_1 \cdot x_2 + m_2 \cdot x_1}{m_1 + m_2}$$

$$\Rightarrow 3 = \frac{2 \times 0 + 3 \times x}{2+3} \quad i.e.\ 15 = 3x \text{ and } x = 5$$

∴ **Co-ordinates of M = (5, 0)** **Ans.**

For point N :

$$P(y) = \frac{m_1 \cdot y_2 + m_2 \cdot y_1}{m_1 + m_2}$$

$$\Rightarrow 2 = \frac{2 \times y + 3 \times 0}{2+3} \quad i.e.\ 10 = 2y \quad i.e.\ y = 5$$

∴ **Co-ordinates of N = (0, 5)** **Ans.**

(ii) ∵ M = (5, 0) and N = (0, 5)

Slope of the line MN $= \frac{5-0}{0-5} = \frac{5}{-5} = \mathbf{-1}$ **Ans.**

(c) A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32 cm. Find the :

(i) radius of the cylinder

(ii) curved surface area of the cylinder. (Take π = 3·1) **[4]**

Solution :

(i) For sphere, radius (r) = 6 cm

and for solid cylinder, height (h) = 32 cm

Let radius of the cylinder be R cm

Clearly, volume of sphere = Volume of cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 h$$

$$\Rightarrow \frac{4}{3} \times \pi \times 6^3 = \pi \times R^2 \times 32$$

$$\Rightarrow 288 = 32R^2 \quad i.e.\ R^2 = \frac{288}{32} = 9 \quad i.e.\ R = 3 \text{ cm}$$

∴ **Radius of the cylinder = 3 cm** **Ans.**

(ii) **Curved surface area of the cylinder** $= 2\pi rh$

$= 2 \times 3{\cdot}1 \times 3 \times 32 \text{ cm}^2$

$= \mathbf{595{\cdot}2\ cm^2}$ **Ans.**

Question **4 :**

(a) The given numbers, K + 3, K + 2, 3K – 7 and 2K – 3 are in proportion. Find K. [3]

Solution :

K + 3, K + 2, 3K – 7 and 2K – 3 are in proportion

$$\Rightarrow \quad (K+3):(K+2) = (3K-7):(2K-3)$$

$$\Rightarrow \quad (K+3)(2K-3) = (K+2)(3K-7)$$

$$\Rightarrow \quad 2K^2 - 3K + 6K - 9 = 3K^2 - 7K + 6K - 14$$

$$\Rightarrow \quad 2K^2 + 3K - 9 = 3K^2 - K - 14$$

$$\Rightarrow \quad 2K^2 - 3K^2 + 3K + K - 9 + 14 = 0$$

i.e. $\quad -K^2 + 4K + 5 = 0$ *i.e.* $K^2 - 4K - 5 = 0$

$\Rightarrow \quad K^2 - 5K + K - 5 = 0$ *i.e.* $K(K-5) + (K-5) = 0$

$\Rightarrow \quad (K-5)(K+1) = 0$ *i.e.* $K = 5$ or $\mathbf{K = -1}$ **Ans.**

(b) Solve for x the quadratic equation $x^2 - 4x - 8 = 0$. Give your answer correct to three significant figures. [3]

Solution :

Comparing $x^2 - 4x - 8 = 0$ with $ax^2 + bx + c = 0$, we get :

$a = 1$, $b = -4$ and $c = -8$

$$b^2 - 4ac = (-4)^2 - 4 \times 1 \times -8$$

$$= 16 + 32 = 48$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{48}}{2 \times 1}$$

$$= \frac{4 \pm 6{\cdot}928}{2}$$

$$= \frac{4 + 6{\cdot}928}{2} \text{ or } \frac{4 - 6{\cdot}928}{2}$$

$$= 5{\cdot}464 \text{ or } -1{\cdot}464$$

$$= \mathbf{5{\cdot}46 \text{ or } -1{\cdot}46}$$

	6·928...
6	48·000000
	36
129	1200
	1161
1382	3900
	2764
13848	113600
	110784
	2816

[*i.e.* $\sqrt{48} = 6{\cdot}928...$] **Ans.**

(c) Use ruler and compass only for answering this question. Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre. Construct two tangents to the circle from the external point P. Measure and write down the length of any one tangent. [4]

Solution :

Steps :

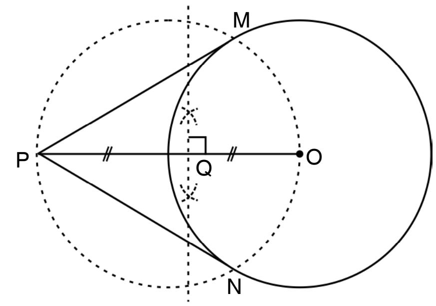
1. Mark a point O, then with O as centre, draw a circle of radius 4 cm.
2. Mark a point P so that OP = 7 cm.

3. Draw perpendicular bisector of OP which meets OP at point Q.
4. With Q as centre, draw a circle with radius = OQ (or PQ).

   Let this circle meet the circle with centre O at points M and N.
5. Join PM and PN.

   PM and PN are the required tangents.

   We know, both the tangents are of equal length and on measuring the length of any of these two tangents, we get the **length of each tangent = 5·7 cm.** **Ans.**



SECTION B (40 Marks)

*(Answer any **four** questions from this Section)*

***Question* 5 :**

(a) There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box.

Find the probability that the number on the disc is :

(i) an odd number **(ii) divisible by 2 and 3 both**

(iii) a number less than 16. **[3]**

Solution :

Here, total number of outcomes = Number of discs = 25

(i) **For an odd number**

Outcomes are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23 and 25

$\Rightarrow$ Number of favourable outcomes = 13

Probability of getting an odd number

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \mathbf{\frac{13}{25}}$$ **Ans.**

(ii) **For the numbers divisible by 2 and 3 both**

Outcomes are 6, 12, 18 and 24

$\Rightarrow$ Number of favourable outcomes = 4

Probability of getting a number divisible by 2 and 3 = $\mathbf{\frac{4}{25}}$ **Ans.**

(iii) **For a number less than 16**

Number of favourable outcomes = 15

$\therefore$ **Required probability** $= \frac{15}{25} = \mathbf{\frac{3}{5}}$ **Ans.**

(b) Rekha opened a recurring deposit account for 20 months. The rate of interest is 9% per annum and Rekha receives ₹ 441 as interest at the time of maturity. Find the amount Rekha deposited each month. **[3]**

Solution :

Given, number of months (n) = 20,

the rate of interest (r) = 9% and

interest = ₹ 441

$$I = P \times \frac{n(n+1)}{2\times 12} \times \frac{r}{100} \qquad \text{[P = amount deposited each month]}$$

$$\Rightarrow \quad 441 = P \times \frac{20\times 21}{2\times 12} \times \frac{9}{100}, \text{ so } P = \frac{441\times 2\times 12}{20\times 21} \times \frac{100}{9} = 280$$

∴ **The amount Rekha deposited each month = ₹ 280** **Ans.**

(c) Use a graph sheet for this question. Take 1 cm = 1 unit along both x and y axes.

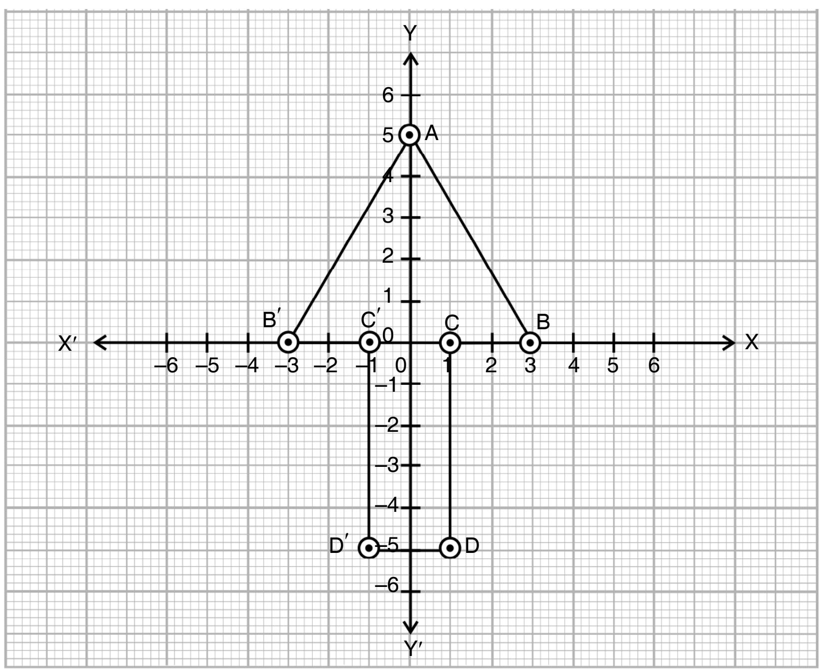
(i) Plot the points : A(0, 5), B(3, 0), C(1, 0) and D(1, –5).

(ii) Reflect the points B, C and D on the y-axis and name them as B′, C′ and D′ respectively.

(iii) Write down the co-ordinates of B′, C′ and D′.

(iv) Join the points A, B, C, D, D′, C′, B′, A in order and give a name to the closed figure ABCDD′C′B′. **[4]**

Solution :



(iii) **B′ = (–3, 0), C′ = (–1, 0)** and **D′ = (–1, –5)** **Ans.**

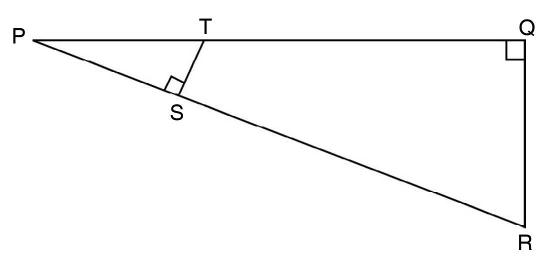
(iv) The closed figure **ABCDD′C′B′A** is like an **arrow head.** **Ans.**

***Question* 6 :**

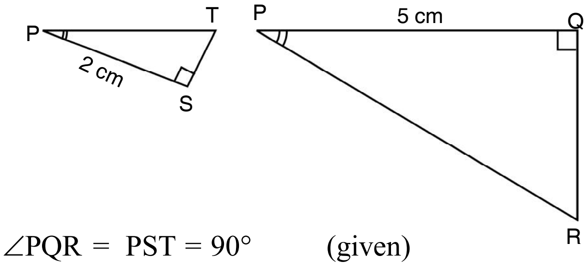
(a) In the given figure, ∠PQR = ∠PST = 90°, PQ = 5 cm and PS = 2 cm.

(i) Prove that ΔPQR ~ ΔPST.

(ii) Find area of ΔPQR : Area of quadrilaterial SRQT. **[3]**



Solution :



(i) $\because$ $\angle PQR = PST = 90°$ (given)

and, $\angle P = \angle P$ (common)

$\therefore$ $\angle PRQ = \angle PTS$ (when two angles of one triangle are equal to two angles of another triangle each to each, their third angles are also equal)

$\Rightarrow$ **$\Delta PQR \sim \Delta PST$** (by A.A.A.) **Hence Proved**

(ii) In similar triangles PQR and PST, PQ = 5 cm and PS = 2 cm are corresponding sides. Since, the ratio between the areas of two similar triangles is equal to the ratio between the squares of their corresponding sides.

$$\Rightarrow \frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PST} = \frac{PQ^2}{PS^2}$$

$$= \frac{5^2}{2^2} = \frac{25}{4}$$

$$\Rightarrow \frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PQR - \text{Area of } \Delta PST} = \frac{25}{25-4}$$

$$\Rightarrow \frac{\text{Area of } \Delta PQR}{\text{Area of quadrilateral SRQT}} = \frac{25}{21}$$

i.e. **Area of ΔPQR : Area of quadrilaeral SRQT = 25 : 21** **Ans.**

(b) The first and last terms of a Geometrical Progression (G.P.) are 3 and 96 respectively. If the common ratio is 2, find :

(i) '*n*' the number of terms of the G.P.

(ii) Sum of the *n* terms. **[3]**

Solution :

Let the first term of G.P. = a

Given, common ratio (r) = 2

(i) $\because$ First term = 3 $\Rightarrow$ $a = 3$

and, last term = 96 $\Rightarrow$ $ar^{n-1} = 96$

i.e. $3 \times 2^{n-1} = 96$ $\Rightarrow$ $2^{n-1} = 32$

$\Rightarrow$ $2^{n-1} = 2^5$ *i.e.* $n - 1 = 5$

and **$n = 6$** **Ans.**

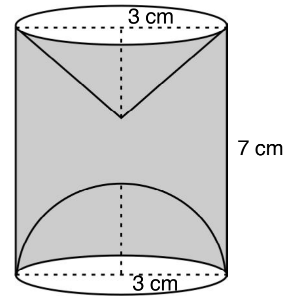
(ii) Sum of n terms $= \dfrac{a(r^n - 1)}{r-1}$

$$= \frac{3 \times (2^6 - 1)}{2-1} = 3 \times (64 - 1) = \mathbf{189}$$ **Ans.**

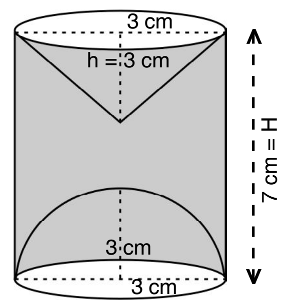
(c) **A hemispherical and a conical hole are scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as as given alongside :**

The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm.

Give your answer correct to the nearest whole number. Take $\pi = \frac{22}{7}$. **[4]**



Solution :



Volume of the remaining solid

= Volume of cylinder – Volume of cone – Volume of hemisphere

$= \pi r^2 H - \frac{1}{3}\pi r^2 h - \frac{2}{3}\pi r^3$

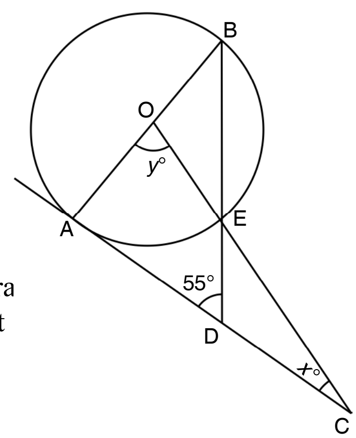
$= \pi[3^2 \times 7 - \frac{1}{3} \times 3^2 \times 3 - \frac{2}{3} \times 3^3]$ cm^3

$= \frac{22}{7} \times 3^2 [7 - 1 - 2]$ cm^3

$= \frac{22}{7} \times 9 \times 4$ cm^3 = 113·14 cm^3 = **113 cm^3** **Ans.**

***Question* 7 :**

(a) **In the given figure AC is a tangent to the circle with centre O. If ∠ADB = 55°, find *x* and *y*. Give reasons for your answer.** **[3]**



Solution :

For *y* :

∠BAD = ∠OAD = 90° [Angle between ra and tangent at point of contact]

In ΔBAD,

∠B + 90° + 55° = 180°

⇒ ∠B = 35°

∠AOE = 2∠B [Angle at centre is twice the angle at remaining circumference]

= 2 × 35°

⇒ $y°$ = 70° *i.e.* ***y* = 70** **Ans.**

For *x* :

In ΔOAC,

$x° + y° + \angle A = 180°$ ⇒ $x° + 70° + 90° = 180°$ *i.e.* ***x* = 20** **Ans.**

(b) The model of a building is constructed with the scale factor 1 : 30.

(i) If the height of the model is 80 cm, find the actual height of the building in metres.

(ii) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model. **[3]**

Solution :

$$\text{Scale factor} = 1 : 30 \quad \Rightarrow \quad k = \frac{1}{30}$$

(i) $\because$ Height of the model = k × Actual height of the building

$\Rightarrow$ $80 \text{ cm} = \frac{1}{30} \times$ Actual height of the building

$\Rightarrow$ **Actual height of the building** = 30 × 80 cm = 2400 cm = **24 m** **Ans.**

(ii) Volume of the tank at the top of the model

$= k^3 \times$ volume of the tank at the top of the building

$= \left(\frac{1}{30}\right)^3 \times 27 \text{ m}^3$

$= \frac{1}{30 \times 30 \times 30} \times 27 \times 100 \times 100 \times 100 \text{ cm}^3$

$= \mathbf{1000 \text{ cm}^3}$ **Ans.**

(c) Given $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ M = 6 *I*, where *M* is a matrix and *I* is unit matrix of order 2 × 2.

(i) State the order of matrix *M*.

(ii) Find the matrix *M*. **[4]**

Solution :

(i) Let order of matrix $M = a \times b$

$\therefore \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6\,I \quad \Rightarrow \quad \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}_{2\times 2} \times M_{a \times b} = 6 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$\Rightarrow$ $a = 2$ and $b = 2$

$\Rightarrow$ **order of matrix *M*** $= a \times b =$ **2 × 2** **Ans.**

(ii) Let matrix $M = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$$\therefore \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix} = 6 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4p + 2r & 4q + 2s \\ -p + r & -q + s \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$\Rightarrow$ $4p + 2r = 6$ ....I $\quad -p + r = 0$ II

$4q + 2s = 0$ ....III and $-q + s = 6$ IV

On solving equations I and II, we get :

$p = 1$ and $r = 1$

And, on solving equations III and IV, we get :

$q = -2$ and $s = 4$

$\therefore$ **Matrix** $\boldsymbol{M} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{1} & \mathbf{4} \end{bmatrix}$ **Ans.**

***Question* 8 :**

(a) The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third terms is 52. Find the first term and the common difference.

Solution :

Let the first term be a and the common difference be d

$\therefore$ First three terms of the A.P. are : a, $a + d$ and $a + 2d$

Given : $a + (a + d) + a + 2d = 42$

$\Rightarrow$ $3a + 3d = 42$

i.e. $a + d = 14 \Rightarrow d = 14 - a$ *i.e.* $a + d = 14 \Rightarrow d = 14 - a$

Given the product of first and the last term is 52

$\Rightarrow$ $a(a + 2d) = 52$ *i.e.* $a^2 + 2ad = 52$

$\Rightarrow$ $a^2 + 2a(14 - a) = 52$ [as, $d = 14 - a$]

$\Rightarrow$ $a^2 + 28a - 2a^2 = 52$

$\Rightarrow$ $a^2 - 28a + 52 = 0$

On solving equation $a^2 - 28a + 52 = 0$, we get : $a = 26$ or $a = 2$

When $a = 26$, $d = 14 - a$

$= 14 - 26 = -12$

And, when $a = 2$, $d = 14 - 2 = 12$

Thus, **if first term = 26, common difference = –12**

and, **if first term = 2, common difference = 12** **Ans.**

Alternative method :

Let the the required three terms in A.P. be $a - d$, a and $a + d$

Given : $(a - d) + a + (a + d) = 42$ and $(a - d)(a + d) = 52$

$\Rightarrow$ $3a = 42$ and $a^2 - d^2 = 52$

$\Rightarrow$ $a = 14$ and $14^2 - 52 = d^2$

$\Rightarrow$ $d^2 = 144$ *i.e.* $d = \pm 12$

For $d = 12$, **first term =** $\boldsymbol{a - d = 14 - 12 = 2}$ **and common difference** $\boldsymbol{d = 12}$ **Ans.**

For $d = -12$, **first term** $\boldsymbol{a - d = 14 + 12 = 26}$ **and common difference** $\boldsymbol{d = -12}$ **Ans.**

(b) The vertices of a ΔABC are A(3, 8), B(–1, 2) and C(6, –6). Find :

(i) Slope of BC.

(ii) Equation of a line perpendicular to BC and passing through A. **[3]**

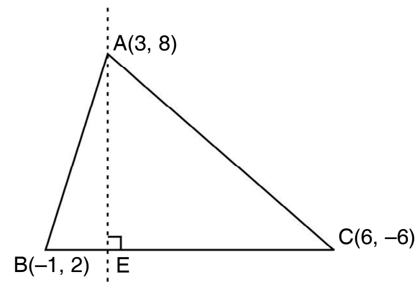
Solution :

(i) Let B(–1, 2) = (x_1, y_2) and

C(6, –6) = (x_2, y_2)

Slope of BC = $\dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-6-2}{6+1} = -\mathbf{\dfrac{8}{7}}$ **Ans.**



(ii) AE is perpendicular to BC and passing through A

$\therefore$ For AE, slope = $\dfrac{7}{8}$ [$\because$ Slope of BC = $-\dfrac{8}{7}$ $\Rightarrow$ slope of its perpendicular AE = $\dfrac{7}{8}$]

and A(x_1, y_1) = (3, 8)

Equation of AE is :

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 8 = \dfrac{7}{8}(x - 3)$

$\Rightarrow \quad 8y - 64 = 7x - 21$ *i.e.* $\mathbf{7x - 8y + 43 = 0}$ **Ans.**

(c) Using ruler and a compass only, construct a semi-circle with diameter BC = 7 cm. Locate a point A on the circumference of the semi-circle such that A is equidistant from B and C. Complete the cyclic quadrilateral ABCD, such that D is equidistant from AB and BC. Measure ∠ADC and write it down. [4]

Solution :

Steps :

1. Draw BC = 7 cm.
2. Draw perpendicular bisector of BC which meets BC at point O.
3. With O as centre and OB (= OC) as radius, draw a semi-circle which meets perpendicular bisector of BC at point A.
4. Since, A lies on the perpendicular bisector of BC, A is equidistant from B and C.
5. D is equidistant from AB and BC.

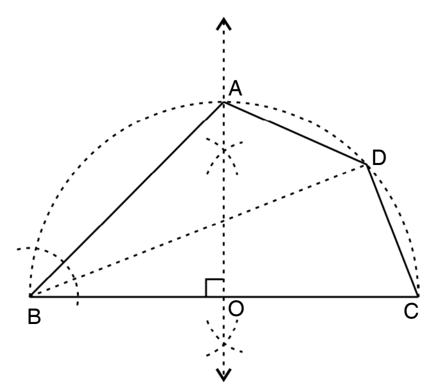
   $\Rightarrow$ D lies on bisector of angle ABC.

   So, draw bisector of angle ABC which meets the semi-circle at point D.

   Join AD and DC.

   $\therefore$ **ABCD is the required cyclic quadrilateral**.

   Also, ∠**ADC = 135°** **Ans.**



***Question* 9 :**

(a) The data on the number of patients attending a hospital in a month are given below. Find the average (mean) number of patients attending the hospital in a month by using the shortcut method. [3]

No. of patients	10-20	20-30	30-40	40-50	50-60	60-70
No. of days	5	2	7	9	2	5

Solution :

No. of patients	No. of days (f)	Class mark (x)	Assumed mean A = 45 $x - A = d$	$f \times d$
10-20	5	15	–30	–150
20-30	2	25	–20	–40
30-40	7	35	–10	–70
40-50	9	45	0	0
50-60	2	55	10	20
60-70	5	65	20	100
	$\Sigma f = 30$			$\Sigma fd = -140$

Average (mean) $= A + \dfrac{\sum fd}{\sum f}$

$= 45 + \dfrac{-140}{30} = 45 - 4{\cdot}67 = \mathbf{40{\cdot}33}$ **Ans.**

(b) Using properties of proportion solve for x, given

$$\frac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = 4.$$ **[3]**

Solution :

Given : $\dfrac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = 4 = \dfrac{4}{1}$

On applying componendo and dividendo, we get :

$$\frac{\sqrt{5x} + \sqrt{2x-6} + \sqrt{5x} - \sqrt{2x-6}}{\sqrt{5x} + \sqrt{2x-6} - \sqrt{5x} + \sqrt{2x-6}} = \frac{4+1}{4-1}$$

$\Rightarrow \quad \dfrac{2\sqrt{5x}}{2\sqrt{2x-6}} = \dfrac{5}{3}$ *i.e.* $\dfrac{\sqrt{5x}}{\sqrt{2x-6}} = \dfrac{5}{3}$ and $\dfrac{5x}{2x-6} = \dfrac{25}{9}$

$\Rightarrow \quad 45x = 50x - 150$ *i.e.* $5x = 150$ and $\mathbf{x = 30}$ **Ans.**

(c) Sachin invests ₹ 8,500 in 10%, ₹ 100 shares at ₹ 170. He sells the shares when the price of each share rises by ₹ 30. He invests the proceeds in 12% ₹ 100 shares at ₹ 125. Find :

(i) the sale proceeds. **(ii) the number of ₹ 125 shares he buys.**

(iii) the change in his annual income. **[4]**

Solution :

(i) $\because$ Investment = ₹ 8,500 and M.V. of each share = ₹ 170

$\therefore$ **Number of shares bought** = $\dfrac{\text{Investment}}{\text{M.V. of each share}} = \dfrac{₹\,8{,}500}{₹\,170} = 50$

Sachin sells each share for ₹ 170 + ₹ 30 = ₹ 200

$\therefore$ **The sale proceeds** = No. of shares sold × S.P. of each share

= 50 × ₹ 200 = **₹ 10,000** **Ans.**

(ii) He invests ₹ 10,000 in shares of M.V. = ₹ 125

Number of ₹ 125 shares be buys = $\dfrac{\text{Investment}}{\text{M.V. of each share}}$

= $\dfrac{₹\,10{,}000}{₹\,125}$ = **80** **Ans.**

(iii) Annual income in 1st case = Dividend on 1 share × no. of shares

= (10% of ₹ 100) × ₹ 50 = ₹ 500

Annual income in 2nd case = Dividend on 1 share × no. of shares

= (12% of ₹ 100) × ₹ 80 = ₹ 960

$\therefore$ **The change in his annual income** = ₹ 960 – ₹ 500

= **₹ 460 (increase)** **Ans.**

***Question* 10 :**

(c) Use graph paper for this question.

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of students	**5**	**9**	**16**	**22**	**26**	**18**	**11**	**6**	**4**	**3**

Draw the ogive and hence, estimate :

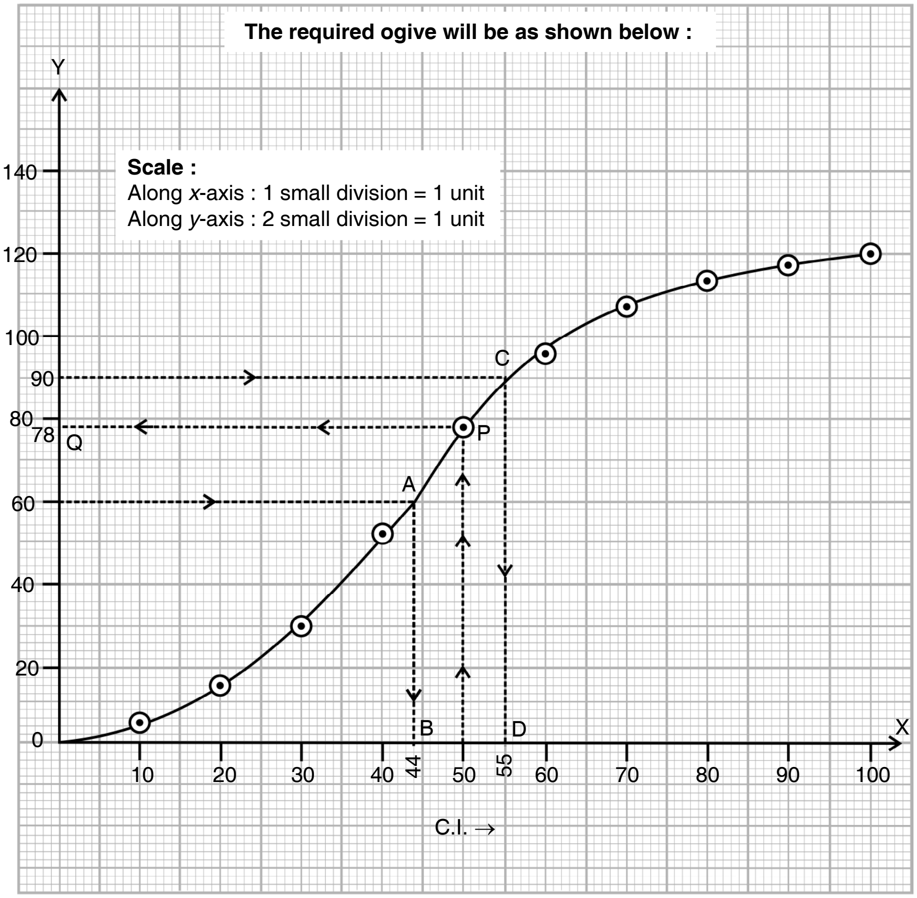
(i) the median marks.

(ii) the number of students who did not pass the test if the pass percentage was 50.

(iii) the upper quartile marks. **[6]**

Solution :

Marks (x)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students (f)	5	9	16	22	26	18	11	6	4	3
($c.f.$)	5	14	30	52	78	96	107	113	117	120



(i) $\because$ $n = 120$

Median $= \left(\frac{120}{2}\right)^{th}$ term $= 60^{th}$ term $=$ **44** **Ans.**

[Through *c.f.* 60, draw a horizontal line which meets graph at point A. Through A, draw a vertical line that meets *x*-axis at point B. Clearly; B = 44].

(ii) $\because$ Pass percentage = 50%

$\Rightarrow$ Pass marks = 50% of 100 = 50

$\therefore$ **The number of students who did not pass**

= Number of students who got marks less than 50

= **78** **Ans.**

[Through 50 on *x*-axis, draw a vertical line that meets graph at pint P. Through P draw a horizontal line that meets *y*-axis at point Q. Clearly Q = 78].

(iii) **The upper quartile** $= \left(\frac{3}{4}\times 120\right)^{th}$ term

$= 90^{th}$ term $=$ **55** **Ans.**

(b) A man observes the angle of elevation of the top of the tower to be 45°. He walks towards it in a horizontal line through its base. On covering 20 m, the angle of elevation changes to 60°. Find the height of the tower correct to 2 significant figures. **[4]**

Solution :

Let the tower be AB.

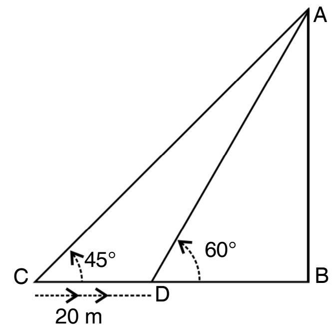
If the first point of observation is C

$\Rightarrow \angle ACB = 45°$

If the second point of observation is D

$\Rightarrow \angle ADB = 60°$

Also, the distance covered = CD = 20 m



In ΔABC, $\tan 45° = \dfrac{AB}{BC} \Rightarrow 1 = \dfrac{AB}{BC}$ and AB = BC I

In ΔABD, $\tan 60° = \dfrac{AB}{BD} \Rightarrow \sqrt{3} = \dfrac{AB}{BD}$ *i.e.* $BD = \dfrac{AB}{\sqrt{3}}$

Now, BC – BD = 20 m $\Rightarrow AB - \dfrac{AB}{\sqrt{3}} = 20$ m

$\Rightarrow \quad AB\left(1-\dfrac{1}{\sqrt{3}}\right) = 20$ m

i.e. $\quad AB \times 0{\cdot}423 = 20$ m

$\Rightarrow \quad \mathbf{AB} = \dfrac{20}{0{\cdot}423}$ m

$= 47{\cdot}28$ m $=$ **47 m** **Ans.**

$1 - \dfrac{1}{\sqrt{3}} = 1 - \dfrac{\sqrt{3}}{3}$

$= 1 - \dfrac{1{\cdot}732}{3}$

$= 1 - 0{\cdot}577 = 0{\cdot}423$

***Question* 11 :**

(a) Using the Remainder Theorem find the remainders obtained when $x^3 + (kx + 8)x + k$ is divided by $x + 1$ and $x - 2$.
Hence, find k if the sum of the two remainders is 1. **[3]**

Solution :

$x + 1 = 0 \Rightarrow x = -1$

Remainder when $x^3 + (kx + 8)x + k$ is divided by $x + 1$

$= \text{The value of } x^3 + (kx + 8)x + k \text{ for } x = -1$

$= (-1)^3 + (k \times -1 + 8) \times -1 + k$

$= -1 + k - 8 + k = 2k - 9$

$x - 2 = 0 \Rightarrow x = 2$

Remainder when $x^3 + (kx + 8)x + k$ is divided by $x - 2$

$= \text{The value of } x^3 + (kx + 8)x + k \text{ for } x = 2$

$= (2)^3 + (k \times 2 + 8) \times 2 + k$

$= 8 + 4k + 16 + k = 5k + 24$

Given, the sum of two remainders = 1

$\Rightarrow (2k - 9) + (5k + 24) = 1$

$\Rightarrow \quad 7k = -14$ and $\mathbf{k = -2}$ **Ans.**

(b) The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. **[3]**

Solution :

Let the required natural numbers be x and $x + 3$

Given $x \times (x + 3) = 810$

$\Rightarrow \quad x^2 + 3x = 810 \quad$ *i.e.* $\quad x^2 + 3x - 810 = 0$

$\Rightarrow \quad x^2 + 30x - 27x - 810 = 0 \quad$ *i.e.* $\quad x(x + 30) - 27(x + 30) = 0$

$\Rightarrow \quad (x + 30)(x - 27) = 0 \quad$ *i.e.* $\quad x = -30$ or $x = 27$

Since, x is a natural number, $x = 27$

Required natural numbers $= x$ and $x + 3$

$= 27$ and $27 + 3 =$ **27 and 30** **Ans.**

Two consecutive natural numbers, which are multiples of 3 differ by 3.

Alternative method :

Let the required natural numbers be $3x$ and $3x + 3$

$\Rightarrow \quad 3x(3x + 3) = 810 \quad$ *i.e.* $\quad 9x^2 + 9x = 810$

$\Rightarrow \quad x^2 + x - 90 = 0 \quad$ *i.e.* $\quad x = -10$ or $x = 9$

$\Rightarrow \quad x = 9$

$\therefore$ Required natural numbers $= 3x$ and $3x + 3$

$= 3 \times 9$ and $3 \times 9 + 3 =$ **27 and 30** **Ans.**

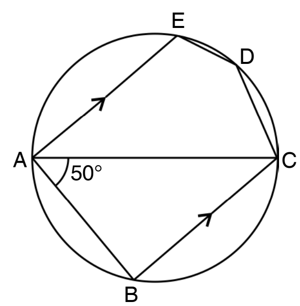
(c) In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC//AE. If $\angle BAC = 50°$, find giving reasons :

(i) $\angle ACB$

(ii) $\angle EDC$

(iii) $\angle BEC$

Hence prove that BE is also a diameter. **[4]**



Solution :

(i) $\because$ AC is diameter

$\Rightarrow \angle ABC = 90° =$ angle of semi-circle

In ΔABC,

$\angle ACB + 50° + 90° = 180° \quad \Rightarrow$ **$\angle ACB = 40°$ Ans.**

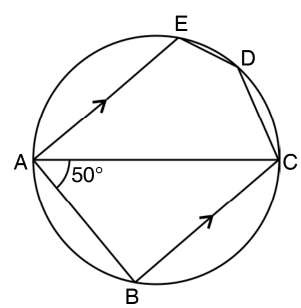
(ii) $\angle EAC = \angle ACB$ (alternate angles)

$= 40°$

In cyclic quadrilateral ACDE,

$\angle EAC + \angle EDC = 180°$

$\Rightarrow 40° + \angle EDC = 180°$ *i.e.* **$\angle EDC = 140°$ Ans.**



(iii) $\angle BEC = \angle BAC$ (angles in same segment)

$=$ **$50°$** **Ans.**

$\angle BAE = 50° + 40°$

$= 90°$

$\Rightarrow \angle BAE$ is the angle of semi-circle

$\Rightarrow$ **BE is a diameter** **Ans.**